Design Tall Masonry Walls

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Outline

- Over view of Single wythe walls
- Present ASD Reinforced Masonry Wall Design Provisions
- Present SD Code provisions Masonry Wall Design Provisions
- Present Tall Wall Design Examples
Single Wythe Walls

Exterior Load Bearing and Non Loadbearing single Wythe Masonry Walls
Exterior Reinforced Masonry RM
Typically subjected to Axial and Flexure Loads

If Load bearing there is a vertical load May be eccentrically Applied
Roof/floor Diaphragm Supports Wall

CODE - COMBINED LOADING – GIVES ASSUMPTIONS
AND SAYS USE MECHANICS – THUS INTERACTION
DIAGRAMS – CODE SECTION 8.3 (ASD) – 9.3 (SD)
Flexure and Axial Load with Reinforced Masonry

  - reinforced masonry walls
  - Present design techniques and aids for design problems involving combinations of flexure and axial force
  - interaction diagrams by hand and by spreadsheet

- Repeat with strength design
Design for Axial Forces and flexure-
(OOP) ASD Interaction Diagrams

Code Section 8.3

COMBINED AXIAL STRESS AND COMPRESSION BENDING STRESS = just add $f_a + f_b$ (no interaction equation)

- Maximum compressive stress in masonry from axial load plus bending must not exceed $(0.45)f'_m$
- Axial compressive stress must not exceed allowable axial stress from Code 8.2.4.1 – Axial forces limited Eq.8-21 or 8-22. Note same result.
- Limit tension stress in reinforcing to $F_s$ – usually 32,000 psi – Compression steel similar if tied (not usual – columns)
Limit $P$ applied $\leq P$ allowable (axial compressive capacity)

Code Equations (8-21) and (8-22)

Slenderness reduction coefficients – same as unreinforced masonry. Compression reinforcing must be tied as per 5.3.1.4 to account for it. i.e. $F_s = 0$ (compression) if not tied.

\[
P_a = (0.25 \, f_m' \, A_n + 0.65 \, A_{st} \, F_s) \left[ 1 - \left( \frac{h}{140r} \right)^2 \right]
\]
for $\frac{h}{r} \leq 99$ (8-21)

\[
P_a = (0.25 \, f_m' \, A_n + 0.65 \, A_{st} \, F_s) \left( \frac{70r}{h} \right)^2
\]
for $\frac{h}{r} > 99$ (8-22)
Axial Compression in Bars Can be accounted for if tied as:

5.3.1.4 Lateral ties:

- (a) Longitudinal reinforcement enclosed by lateral ties at least 1/4 in diameter.

- (b) Vertical spacing of lateral ≤16 longitudinal bar diameters, lateral tie bar or wire diameters, or least cross-sectional dimension of the member.

- (c) Lateral ties shall be arranged such that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a lateral tie with an included angle of not more than 135 degrees. No bar farther than 6 in. (152 mm) clear from such a laterally supported bar. Lateral ties in either a mortar joint or in grout. Where longitudinal bars circular ties shall be 48 tie diameters.

- (d) Lateral ties shall be located one-half lateral tie spacing above the top of footing or slab in any story, one-half a lateral tie spacing below the lowest horizontal reinforcement in beam, ...

- (e) Where beams or brackets frame into a not more than 3 in such beams or brackets.
Allowable – Stress Interaction Diagrams Walls

1. plane sections remain plane after bending
   - shear deformations neglected
   - strain distribution linear with depth
2. neglect all masonry in tension
3. neglect steel in compression unless tied
4. stress-strain relation for masonry is linear in compression
5. stress-strain relation for steel is linear
6. perfect bond between reinforcement and grout
   - strain in grout is equal to strain in adjacent reinforcement
7. grout properties same as masonry unit properties

Assume Solid grouted and

Singly reinforced wall bending out of plane

Bars in center of wall thickness

from Dan Abrams- Masonry Structures Class Notes next 10 Slides
Get equivalent force couple at Centerline of wall thickness

Range “a”:
large P, small M, $e = M/P < t/6$

\[ P_a = 0.5(f_{m1} + f_{m2})A \]
\[ M_a = 0.5(f_{m1} - f_{m2})S \text{ where } S = \frac{bt^2}{6} \]
Axial Force-Moment Interaction Diagram

Out-of-Plane Bending of Reinforced Wall

Range “b”
medium P, medium M, e > t/6, A_s in compression

0.5 < α < 1.0 for section with reinforcement at center

\[ e_m = \frac{t}{2} - \frac{\alpha t}{3} \]

\[ P_b = C_m = \frac{f_{ml}}{2} \alpha t b \]

\[ M_b = C_m e_m \]
Axial Force-Moment Interaction Diagram

Out-of-Plane Bending of Reinforced Wall

Range “c” small P, large M, \( e > t/6 \), \( A_s \) in tension

\[ \alpha < 0.5 \text{ for section with reinforcement at center} \]

\[ e_m = \frac{t}{2} - \frac{\alpha t}{3} \]

\[ \alpha t = kd \]

\[ P_c = C_m - T_s \]

\[ C_m = \frac{f_{ml}}{2} \alpha t b \]

\[ T_s = A_s f_s \]

\[ f_s = \left[ \frac{d - \alpha t}{\alpha t} \right] f_{ml} = \left[ \frac{0.5 - \alpha}{\alpha} \right] f_{ml} \quad \text{for} \quad d = \frac{t}{2} \]

\[ M_c = C_m e_m + T_s \left( d - \frac{t}{2} \right) \]
Axial Force-Moment Interaction Diagram

Out-of-Plane Bending of Reinforced Wall

Range "a"
- Start
- $e=0; M=0$
- $f_{m1}=f_{m2}=F_s$
- $P=F_s A$
- Reduce $f_{m2}$ from $2F_s - F_b$ by increment
- Determine $P$ & $M$ per Range "a"
  - no $f_{m2} = 0$?
  - yes

Range "b"
- Reduce $\alpha$ from 1.0 by increment
- Determine $P$ and $M$ per Range "b"
  - no $\text{is } A_z \text{ in tension}$?
  - yes

Range "c"
- Reduce $\alpha$ from 0.5 by increment
- Determine $P$ and $M$ per Range "c"
  - no $M = 0$?
  - yes
  - no $f_r < F_s$?
  - yes $f_r < F_s$
  - $f_{m1} = F_b$
  - Stop

Tension controlling
Compression controlling
Axial Force-Moment Interaction Diagram
Out-of-Plane Bending of Reinforced Wall

Moment

Axial Force

Range "a"

Range "b"

Range "c"

$f_{m1} = F_b$

$f_{m1} = F_s$

$f_{m2} = 2F_s - F_b$

balanced point

tension controls

compression controls

$F_b$

$F_s$

$F_{m1}$

$F_{m2}$

$F/n$

$F/n$

$F/n$

$F/n$

$F/n$
Allowable – Stress Interaction Diagrams Walls

Example 1 (10.4-2 & 10.4-3 & 10.4-4 MDG)

- 8” CMU with #5 @ 48 in OC
- \( f_m' = 2000 \text{ psi} \) Based on Effective width = 48 in., \( h=16.7 \text{ ft} \)

For \( M = 0 \), \( P \) cutoff - Equation 8-21 or 8-22

\[
\frac{h}{r} = \frac{16.67 \text{ ft} \times 12 \text{ in./ft}}{2.20 \text{ in.}} = 90.9 < 99
\]

\[
P_a = \left( 0.25 f_m' A_n + 0.65 A_{st} F_s \right) \left( 1 - \frac{h}{140r} \right)^2
\]

\[
= \left( 0.25 \times 2,000 \text{ psi} \times (7.63 \text{ in.} \times 48 \text{ in.}) + 0 \right) \left( 1 - \frac{16.67}{140 \times 2.2} \right)^2
\]

\[
= 26,429 \text{ lb} = 26.4 \text{ kips}
\]
## Spreadsheet for calculating allowable-stress M-N diagram for solid masonry wall

**16.67 Ft Wall w/ No. 5 @ 48in (Centered)**  

<table>
<thead>
<tr>
<th><strong>total depth, ( t )</strong></th>
<th>7.625</th>
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<tbody>
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<td><strong>Radius of Gyration, ( r )</strong></td>
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<td><strong>Wall Height, ( h )</strong></td>
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<td><strong>Reduction Factor, ( R )</strong></td>
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<tr>
<td><strong>h/r</strong></td>
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<td><strong>Allowable Axial Stress, ( Fa )</strong></td>
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<tr>
<td><strong>Net Area, ( An )</strong></td>
<td>365.7 ( \text{in}^2 )</td>
</tr>
<tr>
<td><strong>d</strong></td>
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<td><strong>Allowable Axial Compr, ( Pa )</strong></td>
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<tr>
<td><strong>tensile reinforcement, ( As/beff )</strong></td>
<td>0.31 #5 @ 48 Centered</td>
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<tr>
<td><strong>width, beff</strong></td>
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</table>

Because compression reinforcement is not tied, it is not counted.

### Results

<table>
<thead>
<tr>
<th>( k )</th>
<th>( kd )</th>
<th>( fb )</th>
<th>( Cmas )</th>
<th>( fs )</th>
<th>( Axial Force )</th>
<th>( Moment )</th>
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</table>

**Pure compression**

<table>
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<tr>
<th>( Fb = 0.25 f'mR+? )</th>
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</table>

**Axial Force Limits**

<table>
<thead>
<tr>
<th>( Fb )</th>
<th>( 26429 )</th>
</tr>
</thead>
</table>

**NOTE BASED ON 1 ft of Wall and Not EFFECTIVE WIDTH**

\[ n = \frac{E_s}{E_m} = \frac{29,000,000}{900 \times 2000} = 16.11 \]
$$f_b = \frac{n}{d - kd} \times kd = \frac{32,000}{16.11} \times 0.38 = 221 \text{ psi}$$

$$C_{mas} = 1/2 \times f_b \times kd \times b = 1/2 \times 221 \times 0.38 \times 48 = 2,019 \text{ lb}$$

$$P = C_{mas} - (A_s \times F_s) = 2019 - (0.31 \times 32,000) = -7,901 \text{ lb}$$

For $b_{eff}$ for 1 ft of wall = $1/4 \times -7901 = -1975 \text{ lb per foot of wall}$

$$M = C_{mas} \times e_m - T_s \left(d_s - \frac{t}{2}\right) =$$

$$M = 2019 \times \left(\frac{7.625}{2} - \frac{0.38}{3}\right) - T_s(0) = 7,437 \text{ lb.in}$$

For $b_{eff}$ for 1 ft of wall = $1/4 \times 7437 = 1,860 \text{ lb.in per foot of wall}$
\[ F_b = \frac{n f_s}{d - kd} \times kd = \frac{f_s}{3.81 - 1.53} \times 1.53 = 900 \text{ psi} \]

\[ f_s = 21,750 \text{ psi} \]

\[ C_{mas} = \frac{1}{2} \times F_b \times kd \times b = \frac{1}{2} \times 900 \times 1.53 \times 48 = 32,940 \text{ lb} \]

\[ P = C_{mas} - (A_s \times F_s) = 32,940 - (0.31 \times 21,750) = 26,198 \text{ lb} \]

For \( b_{eff} \) for 1 ft of wall = \( 1/4 \times 26,198 = 6549 \text{ lb per foot of wall} \)

\[ M = C_{mas} \times e_m - T_s \left( d_s - \frac{t}{2} \right) = \]

\[ M = 32940 \times \left( \frac{7.625}{2} - \frac{1.53}{3} \right) - T_s(0) = 108,702 \text{ lb.in} \]

For \( b_{eff} \) for 1 ft of wall = \( 1/4 \times 108,702 = 27,200 \text{ lb.in per foot of wall} \)
The table shows values for different parameters. For instance, the entry for $k = 0.311828$, $kd = 1.19$, $fb = 900$, $C_{mas} = 25679$, $fs = -32000$, $Axial\ Force = 3940$, and $Moment = 21931$.

### Calculations

1. **C_{mas}**
   
   \[ C_{mas} = \frac{1}{2} \times F_b \times kd \times b = \frac{1}{2} \times 900 \times 3.81 \times 48 = 82,350 \text{ lb} \]

2. **P**
   
   \[ P = C_{mas} - (A_s \times F_s) = 82,350 - (0.31 \times 0) = 82,350 \text{ lb} \]

   For $b_{eff}$ for 1 ft of wall = $\frac{1}{4} \times 82,350 = 20,587 \text{ lb per foot of wall}$

3. **M**
   
   \[ M = C_{mas} \times e_m - T_s \left( d_s - \frac{t}{2} \right) = \]

   \[ M = 82,350 \times \left( \frac{7.625}{2} - \frac{3.81}{3} \right) - T_s (0) = 209,170 \text{ lb.in} \]

   For $b_{eff}$ for 1 ft of wall = $\frac{1}{4} \times 209170 = 52,300 \text{ lb.in per foot of wall}$
Spreadsheet for calculating allowable-stress M-N diagram for solid masonry wall

### 16.67 Ft Wall w/ No. 5 @ 48in (Centered)

<table>
<thead>
<tr>
<th>physical property</th>
<th>value</th>
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<tbody>
<tr>
<td>total depth, t</td>
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<td>width, beff</td>
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</tr>
</tbody>
</table>

### Notes
- Based on 1 ft of wall and not EFFECTIVE WIDTH

### Calculation

- **Radius of Gyration, r**: 2.20 in
- **Reduction Factor, R**: 0.578
- **Allowable Axial Stress, Fa**: 289 psi
- **Net Area, An**: 365.7 in^2
- **Allowable Axial Compr, Pa**: 26429 lb

### Points controlled by steel

<table>
<thead>
<tr>
<th>k</th>
<th>kd</th>
<th>fb (psi)</th>
<th>Cmas (lb)</th>
<th>fs (psi)</th>
<th>Axial Force (lb)</th>
<th>Moment (lb-in)</th>
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### Points controlled by masonry

<table>
<thead>
<tr>
<th>k</th>
<th>kd</th>
<th>fb (psi)</th>
<th>Cmas (lb)</th>
<th>fs (psi)</th>
<th>Axial Force (lb)</th>
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<td>41175</td>
<td>52327</td>
</tr>
</tbody>
</table>

### Pure compression

- Fb = 0.25 f’mR+?

### Axial Force Limits

- 26429 lb
Ex 1 -

- If P = all dead load
- Check .6D + .6W at mid-height
- Would also have check other load combos
- 0.6D + .6W at mid-height often governs

Per foot of wall analysis
- P at mid-height including weight of wall
- P = 987 lb/ft (0.6 D)
  and M = 7,425 lb.in/ft (0.6D+0.6W)

- You would need to look at other load cases and at the top of the wall.
- Also Check shear – won’t govern.
ASD Interaction Diagram
8 in Wall, 16.67 ft high, #5 @ 48 in. Centered

P, lb per foot of length

M, lb-in per foot of length

0.6D+0.6W
Strength Interaction Diagrams

Code Section 9.3

- for reinforced masonry
  - flexural tensile strength of masonry is neglected
  - equivalent rectangular compressive stress block with maximum stress $0.80 f'_m$, $\beta_1 = 0.80$
  - stress in tensile reinforcement proportional to strain, but not greater than $f_y$

- stress in compressive reinforcement calculated like stress in tensile reinforcement, but neglected unless compressive reinforcement is laterally tied
- Note $\phi = \text{for combined axial and flexure} = 0.9$
Strength Interaction Diagrams...

- vary strain (stress) gradient and position of neutral axis to generate combinations of P and M – using max stress in masonry = 0.8 \( f'_m \) over a compression block and max steel stress = \( f_y \)
- Also limit \( P_u \) applied to \( \leq \phi P_n \) - Eq 9-19 and Eq 9-20

\[
P_n = 0.8(0.80 f'_m (A_n - A_{st}) + A_{st} f_y) \left[ 1 - \left( \frac{h}{140r} \right)^2 \right]
\]

for \( \frac{h}{r} \leq 99 \)  
Eq 9-19

\[
P_n = 0.8(0.80 f'_m (A_n - A_{st}) + A_{st} f_y) \left( \frac{70r}{h} \right)
\]

for \( \frac{h}{r} > 99 \)  
Eq 9-20
Assumptions of Strength Design

- Locate neutral axis based on extreme-fiber strains
- Calculate compressive force C
- $P = C - \sum T_i$ (reduced?)
- $M_{CL} = C(h/2-a/2) \pm \sum T_i y_i$

$\varepsilon_{mu} = 0.0035$ clay
$0.0025$ concrete

$\varepsilon_s \geq \varepsilon_y$

One or more Rebars

$h/2$ Axial Load, $P$
Strength Interaction Example

- vary $c/d$ from zero to a large value
  - when $c = c_{\text{balanced}}$, steel is at yield strain, and masonry is at its maximum useful strain
  - when $c < c_{\text{balanced}}$, $f_s = f_y$
  - when $c > c_{\text{balanced}}$, $f_s < f_y$

- Compute nominal axial force and moment
  - $\phi P_n = \phi \sum f_i$
  - $\phi M_n = \phi \sum f_i x y_i$ (measured from centerline)
  - $\phi = 0.9$

- Can do hand Calc. method like before
Design of Walls for Out-of–Plane Loads: TMS 402 Section 9.3.5
FOR SD MUST ALSO

- **Limit** Maximum reinforcement by 9.3.3.5
- **Check** Nominal shear strength by 9.3.4.1.2
- **Add P Delta Effects to Applied moments**

Three procedures for computing out – of – plane moments and deflections - **Load Side**

- Second – order analysis - **Computer**
- Moment magnification method (**new**)
- Complementary moment method; additional moment from P – δ effects – **Slender wall analysis**
Design of Walls for Out-of-Plane Loads: TMS 402 Section 9.3.5.4.3

- **Moment Magnification all loads**

\[ M_u = \psi M_{u,0} \quad (9-31) \]

\[ \psi = \frac{1}{1 - \frac{P_u}{P_e}} \quad (9-32) \]

\[ P_e = \frac{\pi^2 E_m I_{\text{eff}}}{h^2} \quad (9-33) \]

- \( M_u < M_{\text{cr}} \): \( I_{\text{eff}} = 0.75I_n \)
- \( M_u \geq M_{\text{cr}} \): \( I_{\text{eff}} = I_{\text{cr}} \)
Design of Walls for Out-of–Plane Loads: TMS 402 Section 9.3.5.4.2

- Complementary Moment – Iterative? For $P_u/A_g \leq 0.2 f'_m$

$$M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + P_u \delta_u$$ (9 – 27)

$$P_u = P_{uw} + P_{uf}$$ (9 – 28)

$$\delta_u = \frac{5M_u h^2}{48E_m I_n}$$ (9 – 29)

$$\delta_u = \frac{5M_{cr} h^2}{48E_m I_n} + \frac{5(M_u - M_{cr})h^2}{48E_m I_{cr}}$$ (9 – 30)

- Eq. 9-29 for $M_u < M_{cr}$
- Eq. 9-30 for $M_u \geq M_{cr}$
- Also Limited to $h/t \leq 30$ otherwise $P$ limited to $0.05f'_m A_n$
Design of Walls for Out-of–Plane Loads: TMS 402 Section 9.3.5.4.2

- Or Solve Equations for $M_u$ - 2 eq. 2 unknowns
- $M_u < M_{cr}$

$$M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2}$$

- $M_u \geq M_{cr}$

$$M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + \frac{5M_{cr} P_u h^2}{48 E_m I_n} \left( \frac{1}{I_n} - \frac{1}{I_{cr}} \right)$$

$$M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + \frac{5P_{cr} h^2}{48 E_m I_{cr}}$$
Design of Walls for Out-of–Plane Loads: TMS 402 Section 9.3.5.4.5

- $I_{cr}$

$$I_{cr} = n \left( A_s + \frac{P_u}{F_y} \frac{t_{sp}}{2d} \right) (d - c)^2 + \frac{bc^3}{3}$$

$$c = \frac{A_s F_y + P_u}{0.64 f_m' b}$$
Design of Walls for Out-of–Plane Loads: Design Procedure

- Estimate amount of reinforcement from the following equations.
  - This neglects $P - \delta$ effects. Can estimate increase in moment, such as 10%, for a preliminary estimate of amount of reinforcement.

$$ a = d - \sqrt{d^2 - \frac{2[P_u(d - t/2) + M_u]}{\phi(0.8f'mb)}} $$

$$ A_s = \frac{0.8f'mb/a - P_u / \phi}{f_y} $$

Or take max moment and $d-a/2 \sim 0.9 dA_s F_y = M_u$
DESIGN of REINF MASONRY WALLS Strain Diagrams

- Find $c$ balanced

$$c = d \left( \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} \right)$$

- For $c$ values less than the balanced value steel yields before ult. strain in masonry

- For $c$ above the balanced value steel does not yield at ult. strain in masonry
DESIGN of REINF MASONRY WALLS Strain Diagrams

- For c values less than the balanced value, steel yields before ult. strain in masonry

Note $\beta_1 = 0.8$ in masonry

& $\phi = 0.9$
For c values greater than the balanced value, steel does not yield before ult. strain in masonry.

Note $\beta_1 = 0.8$ in masonry

$\phi = 0.9$
DESIGN of REINF MASONRY WALLS Example 2

WALL

P

e=1.31 in.

4'-4"

21'-0"

CASE A COMPONENTS AND CLADDING WIND LOADS

131.2 or 164.4 lb/ft² (corners)

CASE B COMPONENTS AND CLADDING WIND LOADS

-82.55 or -107.4 (corners) lb/ft²

SEISMIC WALL DESIGN

39.8 lb/ft²

for both Area 4 and 5

-42.7 or - 67.6 (corners) lb/ft

96 lb/ft²

32 lb/ft²
DESIGN of REINF MASONRY WALLS MDG Examples DPC 2 SD

Got SD loads
See example

Then
guessed

$A_s$ to use
Take max.

Moment and back calculated

$A_{sreq}$

$$1.2D + 1.6L_v: + 0.5W_0:
\begin{align*}
P_u &= 1.2 \times (11,900 \ lb + 4 \ ft \ (1,190 \ lb/ft))/4 \ ft \\
    &+ 1.6 \times (11,500 \ lb)/4 \ ft + 0.5 \times (-31,700 \ lb)/4 \ ft \\
    &= 5640 \ lb \ per \ ft \ of \ wall \\
M_u &= (1.31 \ in./2) \times (1.2 \times 11,900 \ lb + 1.6 \times 11,500 \ lb - 0.5 \times 31,700 \ lb \ )/4 \ ft \\
    &\pm 0.5 \times 23,600 \ lb-in. / ft \\
    &= 14,600 \ lb-in. / ft = 1,210 \ lb-ft \ per \ ft \ of \ wall
\end{align*}$$

$$1.2D + 1.0W + 0.5 L_v:
\begin{align*}
P_u &= 1.2 \times (11,900 \ lb + 4 \ ft \ (1,190 \ lb/ft))/4 \ ft \\
    &+ 1.0 \times (-31,700 \ lb)/4 \ ft + 0.5 \times (11,500 \ lb)/4 \ ft \\
    &= -1,490 \ lb \ (uplift) \ per \ ft \ of \ wall \\
M_u &= (1.31 \ in./2) \times (1.2 \times 11,900 \ lb + 1.0 \times (-31,700 \ lb) + 0.5 \times 11,500 \ lb \ )/4 \ ft \\
    &\pm 1.0 \times (-23,600 \ lb-in. / ft) \\
    &= -25,500 \ lb-in. / ft = -2,120 \ lb-ft \ per \ ft \ of \ wall
\end{align*}$$

$$(1.2 + 0.2 S_{ps}) \ D + \rho Q_s \ (Since \ L = 0 \ and \ S = 0)
\begin{align*}
P_u &= 1.2 \times (11,900 \ lb + 4 \ ft \ (1,190 \ lb/ft))/4 \ ft \pm 1.0 \times (3,330 \ lb)/4 \ ft \\
    &= 4,170 \ lb \ per \ ft \ of \ wall \ (negative \ is \ typically \ critical) \\
M_u &= 1.2 \times (1.31 \ in./2 \ (11,900 \ lb) )/4 \ ft + 1.0 \times 21,100 \ lb-in. / ft \\
    &= 23,400 \ lb-in. / ft = 1,950 \ lb-ft \ per \ ft \ of \ wall
\end{align*}$$

$0.9D + 1.0W:
\begin{align*}
P_u &= 0.9 \times (11,900 \ lb + 4 \ ft \ (1,190 \ lb))/4 \ ft + 1.0 \times (-31,700 \ lb))/4 \ ft \\
    &= -4,180 \ lb \ per \ foot \ of \ wall \ (uplift) \\
M_u &= (1.31 \ in./2) \times (0.9 \times 11,900 \ lb + 1.0 \times (-31,700 \ lb))/4 \ ft \pm 1.0 \times 23,600 \ lb-in. / ft \\
    &= -27,040 \ lb-in. / ft = -2,250 \ lb-ft \ per \ ft \ of \ wall
\end{align*}$$
### 21 ft solid 8 in CMU  
#5 @ 16”

Spreadsheet for calculating SD moment-axial force interaction diagram for solid masonry wall reinforcement at mid-depth

<table>
<thead>
<tr>
<th>specified thickness</th>
<th>7.625 in.</th>
<th>Wall Height, h</th>
<th>21.0 feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{mu}$</td>
<td>0.0025</td>
<td>Radius of Gyration, r</td>
<td>2.20 in.</td>
</tr>
<tr>
<td>$f'_m$</td>
<td>2,000 psi</td>
<td>Reduction Factor, R</td>
<td>0.373</td>
</tr>
<tr>
<td>$f_y$</td>
<td>60,000 psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_s$</td>
<td>29,000,000 psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>3.81 in.</td>
<td>Net Area, $A_n$</td>
<td>91.5</td>
</tr>
<tr>
<td>($c/d$)balanced</td>
<td>0.547</td>
<td>Peak Axial Compr. $\phi P_a$</td>
<td>39,265</td>
</tr>
<tr>
<td>tensile reinforcement area</td>
<td>0.2325</td>
<td>No. 5 @ 16 in.</td>
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<tr>
<td>effective width</td>
<td>12 in.</td>
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<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
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</table>

because compression reinforcement is not laterally supported, it is not counted

<table>
<thead>
<tr>
<th>$c/d$</th>
<th>$c$</th>
<th>$C_{mas}$</th>
<th>$f_s$</th>
<th>$M_n$, lb-in</th>
<th>$P_a$, lb</th>
<th>$\Phi M_n$, lb-in</th>
<th>$\Phi P_a$, lb</th>
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<td>0.32</td>
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<td>-60,000</td>
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<td>4,194</td>
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<td>0.33</td>
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<td>57,492</td>
<td>4,826</td>
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<tr>
<td>0.4</td>
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<td>-60,000</td>
<td>67,447</td>
<td>8,513</td>
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<td></td>
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<tr>
<td>Balanced point</td>
<td>0.547169811</td>
<td>2.084717</td>
<td>32,021</td>
<td>-60,000</td>
<td>85,810</td>
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<td>Points controlled</td>
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<td></td>
<td></td>
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<tr>
<td>0.90</td>
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<td>-8,056</td>
<td>115,701</td>
<td>45,717</td>
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<tr>
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<td>4.191</td>
<td>64,374</td>
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<td>123,758</td>
<td>57,936</td>
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<td>0</td>
<td>329</td>
<td>131,674</td>
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<td></td>
</tr>
</tbody>
</table>

Maximum 0.05$f'_m$ | 9,150 |
Maximum 0.2 $f'_m$ | 36,600 |

$P_a$ | 39,265
Develop ID

steel controls

\[ C_{\text{mas}} = 0.8 \times c \times 0.8 f'_m \times b = 0.8 \times 1.143 \times 0.8 \times 2000 \times 12 = 17,566 \text{ lb per foot of wall} \]

\[ P = C_{\text{mas}} - (A_s \times F_y) = (17,556 - (0.2325 \times 60,000)) = 3606 \text{ lb} \]

\[ \phi P_n = 0.9 \times 3,606 \text{ lb} = 3,245 \text{ lb per foot of wall} \]

\[ M = T (d - \frac{h}{2}) + C_{\text{mas}} \left( \frac{h}{2} - \frac{\beta_1 c}{2} \right) = \]

\[ M = 60000 \times 0.02325 \times \left( \frac{7.625}{2} - \frac{7.625}{2} \right) + 17655 \left( \frac{7.626}{2} - \frac{(0.8)1.143}{2} \right) = \]

\[ = 58,872 \text{ lb.in per foot of wall} \]

\[ \phi M = 52,985 \text{ lb.in per foot of wall} \]
### Spreadsheet for calculating SD moment-axial force interaction diagram for solid masonry wall reinforcement at mid-depth

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</tr>
</tbody>
</table>

- **Tensile reinforcement area**: 0.2325, **No. 5 @ 16 in.**
- **Effective width**: 12 in.
- \( \phi \): 0.9

Because compression reinforcement is not laterally supported, it is not counted.

<table>
<thead>
<tr>
<th>( \frac{c}{d} )</th>
<th>( c )</th>
<th>( C_{mas} )</th>
<th>( f_{s} )</th>
<th>Moment ( \Phi M_{n} ), lb-in</th>
<th>Axial Force ( \Phi P_{n} ), lb</th>
</tr>
</thead>
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<tr>
<td>Points controlled</td>
<td>0.01</td>
<td>0.0381</td>
<td>585</td>
<td>-60,000</td>
<td>1,969</td>
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<td>146,304</td>
<td>0</td>
<td>329</td>
</tr>
</tbody>
</table>

**Maximum** 0.05\( f'_{m} \) 9,150  
**Maximum** 0.2 \( f'_{m} \) 36,600  
**Reduction Factor, \( R \)** 0.373
Develop ID

Masonry controls

<table>
<thead>
<tr>
<th>c/d</th>
<th>c</th>
<th>$C_{mp,\text{f}}$</th>
<th>$f_{\text{s}}$</th>
<th>Moment</th>
<th>Axial Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.90</td>
<td>3.429</td>
<td>52,669</td>
<td>-8,056</td>
<td>115,701</td>
</tr>
</tbody>
</table>

$C_{\text{mas}} = 0.8 \times c \times 0.8 f'_{\text{m}} \times b = 0.8 \times 3.429 \times 0.8 \times 2000 \times 12 = 52,669 \text{ lb per foot of wall}$

$\varepsilon_s = \varepsilon_{\text{mu}} \left( \frac{d - c}{c} \right) = 0.0025 \left( \frac{3.81 - 3.429}{3.429} \right) = 0.000278$

$f_s = 29,000,000 \times 0.000278 = 8,056 \text{ psi}$

$P = C_{\text{mas}} - (A_s \times F_s) = (52,669 - (0.2325 \times 8056)) = 50,796 \text{ lb}$

$\phi P_n = 0.9 \times 50796 \text{ lb} = 45,716 \text{ lb per foot of wall}$

$M = T \left( d - \frac{h}{2} \right) + C_{\text{mas}} \left( \frac{h}{2} - \frac{\beta_1 c}{2} \right) =$

$M = 8,056 \times 0.02325 \times \left( \frac{7.625}{2} - \frac{7.625}{2} \right) + 52669 \left( \frac{7.626}{2} - \frac{(0.8)3.429}{2} \right) =$

$= 128,428 \text{ lb.in per foot of wall}$

$\phi M = 115,600 \text{ lb.in per foot of wall}$
Develop ID

Get all the points and cut off any above max P

Also calc.
0.05$f'_{_m}$An
and
0.2$f'_{_m}$An

These are cutoffs for P-$\Delta$ methods

### Spreadsheet for calculating SD moment-axial force interaction diagram for solid masonry wall

- **21 ft solid 8 in CMU** #5 @ 16''
- Wall Height, $h$ = 21.0 feet

#### Reinforcement at mid-depth
- Specified thickness: 7.625 in.
- $e_{mu}$: 0.0025
- $f'_{m}$: 2,000 psi
- $f_y$: 60,000 psi
- $E_s$: 29,000,000 psi
- $d$: 3.81 in.
- $(c/d)_{balanced}$: 0.547
- Tensile reinforcement area: 0.2325
- Effective width: 12 in.
- $\phi$: 0.9

#### Additional Variables
- **Radius of Gyration, $r$**: 2.20 in.
- **Reduction Factor, $R$**: 0.373
- **$\mu_0$**: 0.0025
- **$f'_{m}$**: 9,150 psi
- **$f'_{m}$**: 36,600 psi

#### Points controlled by steel

<table>
<thead>
<tr>
<th>$c/d$</th>
<th>$c$</th>
<th>$C_{mas}$</th>
<th>$f_s$</th>
<th>$\Phi M_n$, lb-in</th>
<th>$\Phi P_n$, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0381</td>
<td>585</td>
<td>-60,000</td>
<td>1,969</td>
<td>-12,028</td>
</tr>
<tr>
<td>0.15</td>
<td>0.5715</td>
<td>8,778</td>
<td>-60,000</td>
<td>28,283</td>
<td>-4,655</td>
</tr>
<tr>
<td>0.20</td>
<td>0.762</td>
<td>11,704</td>
<td>-60,000</td>
<td>36,918</td>
<td>-2,021</td>
</tr>
<tr>
<td>0.30</td>
<td>1.143</td>
<td>17,556</td>
<td>-60,000</td>
<td>52,985</td>
<td>3,246</td>
</tr>
<tr>
<td>0.32</td>
<td>1.21158</td>
<td>18,610</td>
<td>-60,000</td>
<td>55,707</td>
<td>4,194</td>
</tr>
<tr>
<td>0.33</td>
<td>1.2573</td>
<td>19,312</td>
<td>-60,000</td>
<td>57,492</td>
<td>4,826</td>
</tr>
<tr>
<td>0.40</td>
<td>1.524</td>
<td>23,409</td>
<td>-60,000</td>
<td>67,447</td>
<td>8,513</td>
</tr>
</tbody>
</table>

#### Balanced point

- $(c/d)_{balanced}$: 0.547
- Net Area, $A_n$: 91.5
- **Peak Axial Compr, $\Phi P_a$**: 39,265

#### Points controlled by masonry

<table>
<thead>
<tr>
<th>$c/d$</th>
<th>$c$</th>
<th>$C_{mas}$</th>
<th>$f_s$</th>
<th>$\Phi M_n$, lb-in</th>
<th>$\Phi P_n$, lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>3.048</td>
<td>46,817</td>
<td>-18,125</td>
<td>109,261</td>
<td>38,343</td>
</tr>
<tr>
<td>0.90</td>
<td>3.429</td>
<td>52,669</td>
<td>-8,056</td>
<td>115,701</td>
<td>45,717</td>
</tr>
<tr>
<td>1.10</td>
<td>4.191</td>
<td>64,374</td>
<td>0</td>
<td>123,758</td>
<td>57,936</td>
</tr>
<tr>
<td>1.30</td>
<td>4.953</td>
<td>76,078</td>
<td>0</td>
<td>125,390</td>
<td>68,470</td>
</tr>
<tr>
<td>1.60</td>
<td>6.096</td>
<td>93,635</td>
<td>0</td>
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<td>84,271</td>
</tr>
<tr>
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<td>105,339</td>
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<td>94,805</td>
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<td>2.10</td>
<td>8.001</td>
<td>122,895</td>
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<td>110,606</td>
</tr>
<tr>
<td>2.50</td>
<td>9.525</td>
<td>146,304</td>
<td>0</td>
<td>329</td>
<td>131,674</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$M$</th>
<th>$P_{max}$</th>
<th>$P_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>39,265</td>
<td>125,390</td>
</tr>
<tr>
<td>0.2 $f'_{m}$</td>
<td>36,600</td>
<td>105,000</td>
</tr>
</tbody>
</table>
SD Interaction Diagram
8 in. solid CMU wall, $f_m'=2,000$ psi
No.5 bars @ 16 in
Add P-D effects to applied Moments – Complementary Method or Moment Magn. methods

Complementary Method

\[ M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + P_u \delta_u \]  

(9 – 27)

- Or Solve Equations for \( M_u \) - 2 eq. 2 unknowns
- \( M_u < M_{cr} \)

\[ M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} \]

\[ M_u = \frac{5P_u h^2}{1 - \frac{48E_m I_n}{48E_m I_n}} \]

- \( M_u \geq M_{cr} \)

\[ M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + \frac{5M_{cr} P_u h^2}{48E_m} \left( \frac{1}{I_n} - \frac{1}{I_{cr}} \right) \]

\[ 1 - \frac{5P_u h^2}{48E_m I_{cr}} \]
Add P-D effects to applied Moments – Complementary Method or Moment Magn. methods

Moment Mag. Method

\[ M_u = \psi M_{u,0} \quad (9 - 31) \]

\[ \psi = \frac{1}{1 - \frac{P_u}{P_e}} \quad (9 - 32) \]

\[ P_e = \frac{\pi^2 E_m I_{eff}}{h^2} \quad (9 - 33) \]

1.2D+1.6Lr+0.5W load combination

\[ M_u = \text{first order moment} = 14.6 \text{ kips.in/ft of wall} \]
Add P-D effects to applied Moments – Complementary Method or Moment Magn. methods

Moment Mag. Method – Get \( I_g \) and \( I_{cr} \)

\[
I_g = \frac{bt^3}{12} = \frac{12 \text{ in.} \cdot (7.63 \text{ in.})^3}{12} = 443 \text{ in}^3
\]

\[
n = \frac{E_s}{E_m} = \frac{29,000,000 \text{ psi}}{900(2,000 \text{ psi})} = 16.11
\]

\[
A_s = \frac{0.31 \text{ in.}^2 (12 \text{ in.})}{16 \text{ in.}} = 0.232 \text{ in.}^2
\]

\[
c = \frac{A_s f_y + P_u}{0.64 f'_m b} = \frac{0.232 \text{ in.}^2 (60,000 \text{ psi}) + 5,640 \text{ lb}}{0.64(2,000 \text{ psi})(12 \text{ in.})} = 1.273 \text{ in.} \quad \text{TMS 402 Equation 9 - 35}
\]

\[
I_{cr} = n \left( A_s + \frac{P_u}{f_y} \frac{t_{sp}}{2d} \right) \left( d - c \right)^2 + \frac{bc^3}{3}
\]

Equation 9 - 34
Add P-D effects to applied Moments – Complementary Method or Moment Magn. methods

\[ I_{cr} = 16.1 \left( 0.232 \text{ in.}^2 + \frac{5,640 \text{ lb}}{60,000 \text{ psi}} \times \frac{7.63 \text{ in.}}{2(3.8 \text{ in.}^2)} \right) (3.8 \text{ in.} - 1.27 \text{ in.})^2 = 33.9 \text{ in}^4 / \text{ ft} \]

From TMS 402 Table 9.1.9.2, \( f_r = 163 \text{ psi} \)

\[ M_{cr} = \left( f_r + \frac{P}{A_g} \right) S = \left( 163 \text{ psi} + \frac{5,640 \text{ lb}}{12 \text{ in.} \times (7.63 \text{ in.})} \right) (12 \text{ in.})(7.63 \text{ in.})^2 \right) / 6 = 26,150 \text{ lb.in/ft} \]

As \( M_u = 14,600 \text{ lb-in} \) is less than \( M_{cr} = 26,150 \text{ lb-in} \) then

\[ I_{eff} = 0.75 \ (I_n) \]
Add P-D effects to applied Moments – Complementary Method or Moment Magn. methods

\[ P_e = \frac{\pi^2 E_m I_{eff}}{h^2} = \frac{\pi^2 900 (2,000 \text{ psi})(443 \text{ in}^4)(0.75)}{(21 \text{ ft} \times 12 \text{ in.}/\text{ft})^2} = 93,000 \text{ lb} \]

\[ \psi = \frac{1}{1 - \frac{P_u}{P_e}} = \frac{1}{1 - \frac{5,640 \text{ lb}}{93,000 \text{ lb}}} = 1.06 \]

\[ M_u = 1.06 (14.6 \text{ kip.in/ft}) = 15.5 \text{ kip.in/ft which is still below the capacity envelope} \]

Check other Load cases
SD Interaction Diagram

8 in. solid CMU wall, $f'_m=2,000$ psi
No.5 bars @ 16 in
DESIGN of REINF MASONRY WALLS - Out of Plane loads – Axial and flexure

- Must check Max Reinforcing 9.3.3.5 Comm.

\[
\rho_{\text{max}} = \frac{0.64 f'_m \left( \frac{\varepsilon_{\mu}}{\varepsilon_{\mu} + \alpha \varepsilon_y} \right) - \frac{P}{bd}}{f_y}
\]

\( P \) load \( D + 0.75 L + 0.525 Q_e \) load combination and \( \alpha = 1.5 \) for walls loaded OOP. The force per foot of wall is:

\( P = (11,900 \text{ lb/4} + 1,190 \text{ lb}) + 0.75 \times 11,500 \text{ lb/4} + 0.525 \times 3,330 \text{ lb/4} = 6,760 \text{ lb} \)
DESIGN of REINF MASONRY WALLS - Out of Plane loads – Axial and flexure

Must check Max Reinforcing 9.3.3.5 Comm.

\[
\rho_{\text{max}} = \frac{0.64 \times 2,000 \text{ psi} \times \left( \frac{0.0025}{0.0025 + 1.5 \times 0.00207} \right) - \frac{6,760 \text{ lb}}{12 \text{ in.} \times 3.81 \text{ in.}}}{60,000 \text{ psi}} = .00705
\]

This would allow an area of steel of up to \( As = 0.00705 \times 12 \text{ in.} \times 3.81 \text{ in.} = 0.322 \text{ in}^2 \) per foot of wall, which is more than 0.233 in\(^2\)/ft provided using No. 5 bars at 16 in. on center.
DESIGN of REINF MASONRY WALLS - Out of Plane loads – Axial and flexure

- Should check shear but never a problem out of plane for LB and non LB walls.
DESIGN of REINF MASONRY WALLS
- Out of Plane loads – Axial and flexure -

How high can you go?
For this Loading

$P = 1000 \text{ lb/ft } L, 500 \text{ lb/ft } D$

$h = ?$

Roof/floor Diaphragm Supports Wall

$30 \text{ psf}$

$e = 2 \text{ in}$
8" CMU

ASD

H = 30 ft

# 6 bars at 16” on center
d = 3.81”

fully grouted 8” CMU

f’_m = 2000 psi

ASD design
How high - 8” CMU?

ASD H = 39 ft

# 6 bars at 16” on Staggered d = 5”

fully grouted 8” CMU

$f_m = 2000 \text{ psi}$

ASD design
8" CMU

SD

$H = 30 \text{ ft}$

#6 bars at 16" on center
d = 3.81"

fully grouted
8" CMU

$f'_m = 2000 \text{ psi}$

SD design

Be careful with $f_r$ – Very slender walls - use complementary Method and account for P/A for Cracking moment
Try 12” CMU

ASD Interaction Diagram
8 in Wall, 44.5 ft high, #6 @ 16
Centered

ASD H = 44.5 ft
# 6 bars at 16” on
Centered  \( d = 5.81” \)

fully grouted 12”
CMU

\( f_m' = 2000 \text{ psi} \)

ASD design

No. 6 bars  Staggered (typ.)

\( d = 5.81” \)

16”
Try 12” CMU

ASD Interaction Diagram
8 in Wall, 55 ft high, #6 @ 16
Staggard d = 8.75

ASD H = 55 ft

# 6 bars at 16” on
Centered  d = 5.81”

fully grouted 12”
CMU

f’ m = 2000 psi

ASD design
Summary

- Over view of Single wythe walls
- Presented ASD Reinforced Masonry Wall Design Provisions
- Presented SD Code provisions Masonry Wall Design Provisions
- Presented Wall Design Examples
THANK YOU!

QUESTIONS?